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Physics in Perspective

Gravity as Spacetime Curvature*

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Einstein's general theory of relativity conceives the phenomena of gravity as manifestations of the curvature of the spacetime manifold in which physical events take place. I sketch the line of thought that led Einstein to this conception, and I briefly discuss proposals by Jeffreys and Feynman for retaining Einstein's gravitational field equations while discarding their purportedly geometrical meaning.

Key words: Gravity; spacetime; spacetime curvature; general relativity; Einstein; Feynman; Harold Jeffreys.

Since the advent of Albert Einstein's theory of gravitation in the fall of 1915,[1] the question is often asked: "Why should gravity be thought of as curved spacetime?" This question can be understood as follows: There exists "out there," independently of our modes of thought, a definite natural force we call "gravity" or a natural kind of effects we call "gravitational," which, when we come to think of them, we are bound to conceive as a consequence of the fact that the world we live in embodies the mathematical structure known as "curved spacetime." The question asks why we are so bound; specifically, what results of scientific research constrain us to think in this way.

This reading of the question, though seemingly natural, entails, however, in my view, that it is either impertinent or trivial. The term "gravity," which is the anglicized form of Latin *gravitas*, *i.e.*, "heaviness," has had its extension drastically changed with successive changes in physical theory. The family of gravitational phenomena, which in Antiquity would have comprised only the fall of heavy bodies on earth and the downward pressure exerted by them when stopped from falling, was enlarged by Isaac Newton to encompass the motion of the heavenly bodies. Newton's theory of universal attraction still determines, I dare say, the meaning of "gravity" and "gravitational" in ordinary English; but when we think of gravity as curved spacetime, governed by the Einstein field equations, we automatically add to the family such seemingly disparate phenomena as changes in the motion of binary pulsars due to the emission of gravitational waves, apparent duplication of light

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sources by so-called gravitational lenses, and the long-term cooling down of the background thermal radiation discovered by Arno A. Penzias and Robert W. Wilson. In view of the ambiguity of the word "gravity," the question, "Why should gravity be thought of as curved spacetime?" splits into many. On the one hand, neither Aristotelian nor Newtonian gravity are thought of as curved spacetime, so the question does not apply to them. On the other hand, Einstein's general relativity theory conceives gravitational phenomena as manifestations of spacetime curvature, so it is no wonder that gravity in Einstein's sense must be thought of as stated. Applied to Einstein's concept, the question is pertinent but trivial.

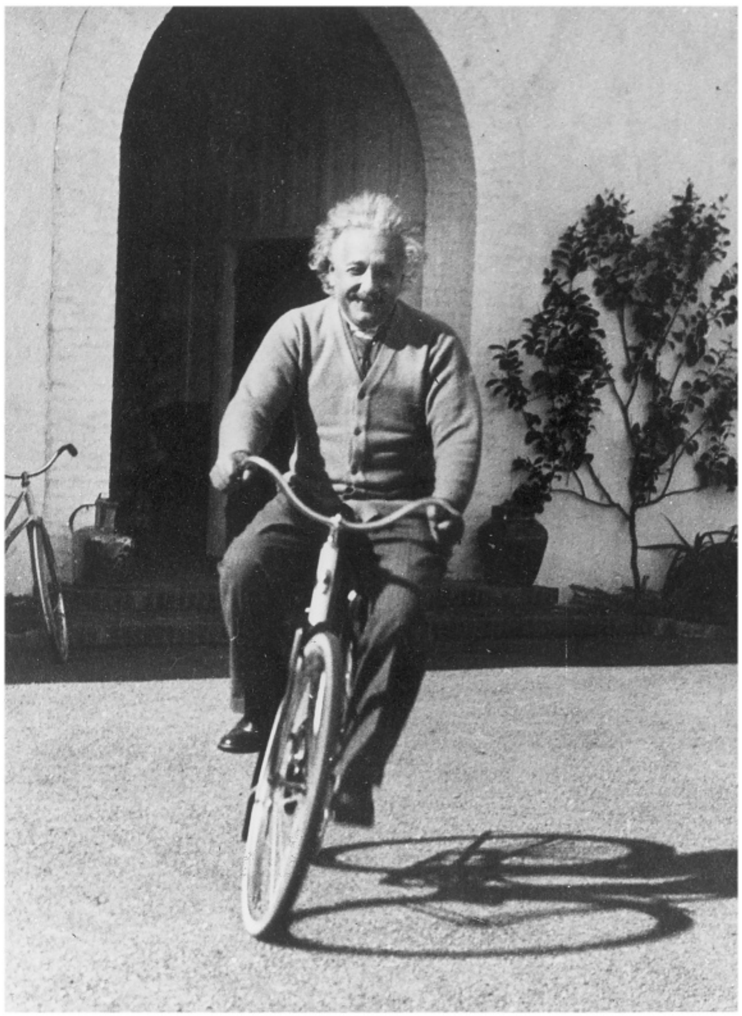

Fig. 1. Albert Einstein riding a bicycle on February 6, 1933, at home of friends in Santa Barbara, California. Courtesy of the Archives, California Institute of Technology. *Albert Einstein™ licensed by the Hebrew University of Jerusalem. Represented by the Roger Richman Agency, Inc., Beverly Hills, CA.*

The question, however, allows a different reading, which is not liable to this criticism. We may paraphrase it thus: Why should the physical phenomena that Einstein's theory classifies as gravitational be considered to be such *in Einstein's sense of the term*? Why ought we to think of them as forming a natural kind, and to think so precisely for the reason given by general relativity, namely, that, in one way or another, they exhibit the presence of spacetime curvature? Note that Einstein's initial work on gravity concerned the phenomena grouped under Newton's concept. Additional phenomena, such as the so-called gravitational retardation of clocks and the deflection of starlight grazing the sun, as well as the others I mentioned earlier, accrued to the extension of Einstein's concept on the strength of his theory, which predicted such phenomena and in effect put researchers on the lookout for many of them. From this perspective our question has to do with conceptual change: Why did Einstein judge it necessary to reconceive the phenomena covered by Newton's concept of gravity and to understand them as due to the curvature of spacetime? But it is also a question of adequacy: Why should Einstein's classification and understanding of gravitational phenomena be preferred, instead of other available alternatives? What—if anything—compels us to think that Einstein's concept of gravity applies to the phenomena it is intended to grasp? In what follows I shall mostly ignore this second aspect of the question and deal only with the first.

Einstein began working on a new theory of gravity in 1907. What prompted him to do so? In my opinion, this question has a very simple answer: Newton's law of gravity clashes head-on with the general principles of physics Einstein put forward in 1905, the core principles of what we now call special relativity. In his paper, "On the electrodynamics of moving bodies,"[2] Einstein postulated the thoroughgoing equivalence of inertial frames of reference ("Relativity Principle") and the constancy of the vacuum speed of light in any such frame, and proved that the joint assertion of these two postulates entails the Lorentz invariance of the laws of physics. Newton's law of gravity is not Lorentz invariant and therefore had to be revised for inclusion in the new physics. This was noted by Henri Poincaré in a paper of 1906,[3] in which he independently reached conclusions practically indistinguishable from Einstein's. In its final section, Poincaré sketched a Lorentz-invariant theory of gravity. There is no need to comment on this theory here, but I shall note two steps that Poincaré took on his way to it.

First, in order to handle the problem of gravity, Poincaré represented physical phenomena in a four-dimensional manifold charted by coordinates $\langle x, y, z, t \rangle$ (space coordinates x, y, z in light-seconds, time coordinate t in seconds) and proved the Lorentz invariance of the quadratic form

$$x^2 + y^2 + z^2 - t^2 \tag{1}$$

As is well known, this approach was soon taken up by Hermann Minkowski,[4] whose chronogeometric reading of special relativity was absolutely essential for the formulation of Einstein's theory of gravity or general relativity.

Second, Poincaré resolved that, in choosing among Lorentz-invariant candidates for a new law of gravity, he ought to pick the one which, for small relative speeds of the interacting bodies, differed least from Newton's. For, he said, "astronomical observations do not appear to show any significant deviation from Newton's law."[5] This remark indicates that Poincaré, the foremost world expert in celestial mechanics, did not think much of the discrepancy between the predicted and the observed value of Mercury's secular perihelion advance (amounting to almost eight parts in a thousand).

Poincaré's attitude may be contrasted with Einstein's, who wrote to a friend on Christmas eve, 1907, that he was working on a new theory of gravity which, he expected, would account for "still unexplained secular variations" of Mercury's perihelion.[6] Einstein's Christmas message lends color to the familiar empiricist view, according to which progress in theoretical physics follows in some such simple-minded way upon every petty failure of theory-based predictions. Einstein's interest in the perihelion anomaly is confirmed by the fact that, when he finally had a solution for it, based on the theory of gravity he published on November 11, 1915,[7] he rushed to publish it, a week later,[8] before even being sure that the theory was worth pursuing.* Nevertheless, I believe that to him the perihelion anomaly was only a neat test for the theory of gravity he was reaching for in pursuit of much broader interests. It surely would be a good thing if, on this particular point, the new theory did better than Newton's. But Einstein was ready to countenance one that did not. Thus, he did not hesitate to publish, in 1913, the theory of gravity he developed in collaboration with Marcel Grossmann, which failed to solve the anomaly of Mercury.[9] Indeed, when he presented this theory to the assembly of German natural scientists in Vienna that same year, he paid tribute to Newton in even stronger terms than Poincaré. Newton's "law of the interaction between two gravitating point-masses [. . . has] proved to be so exactly right that, from the standpoint of experience, there is no decisive ground for doubting [its] strict validity."[10] Whatever we might think of Mercury's anomaly as a driving motive of Einstein's research, it cannot, in any sense, be said to have guided it. Einstein did not try out corrections that could better fit the observations** but went straight for a grand principle that would give him a grip on the very root of gravity. The one he found, the Equivalence Principle, was included also, though only as a corollary, in Newton's theory, but Einstein wielded it in a way conducive to a novel—and wholly unexpected—understanding of gravitational phenomena.

To show how this worked I must recall the essentials of the Newtonian conception. By Newton's First and Second Laws of Motion, a body rests or moves with constant speed in a straight line unless it is accelerated by an external force,

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* Einstein abandoned that theory on November 25, when he published the field equations of general relativity (ref. 1). These equations, however, yield the same solution of the perihelion anomaly, inasmuch as they agree with the equations of November 11 on the empty interstellar space through which the planets circulate.

** Like the astronomer A. Hall, who proposed to substitute $1/r^{2.00000016}$ for $1/r^2$ in Newton's law! (*Astronomical Journal* **14** (1894), 49; I owe this reference to W. Rindler).

the acceleration due to an external force being proportional to it and inversely proportional to the body's quantity of matter or *mass*. However, since Galileo the main tenet of modern gravitational physics has been that acceleration due to gravity is the same for all bodies. Newton accepted this tenet; indeed he verified it in experiments with pendulums made from different materials, which he says were accurate to 1 part in 1,000.[11] So Newton solved the apparent conflict with his Laws of Motion by assuming that the action of gravitational force on any body is directly proportional to the body's mass. Thus the *inertia* with which the body resists gravitational acceleration is exactly compensated by its liability to gravitate, the gravitational analogue of *charge*.

In the context of special relativity Newton's handling of this matter becomes highly questionable. For a body's inertia is not measured by a Lorentz-invariant quantity, and increases with its speed relative to the inertial frame to which it is referred. Does the same hold true of gravitational charge? Max Planck surmised that it did not, and that the empirically established equality between "inertial" and "ponderable" mass was in fact no more than a useful approximation.[12] However, Einstein decided, about the same time, to abide by the null result of Newton's pendulum experiments, which Roland von Eötvös[13] had improved *ca*. 1890 to 1 part in 100 million, using a torsion balance (the null result was subsequently confirmed by V. B. Braginsky and V. I. Panov[14] to 1 part in 1 trillion). However, Einstein did not explain it, like Newton, by the self-effacement of a physical quantity acting against itself, but by what he saw as a natural extension of the Relativity Principle. He could indeed reflect that this principle, in its original version, restricted to inertial frames, had enabled him to account for the null result of the Michelson-Morley experiment without resorting to compensatory physical effects like the Lorentz-Fitzgerald contraction.*

The Equivalence Principle is usually stated in Einstein's writings as an application of relativity to uniformly accelerated frames of reference. Those statements may be paraphrased as follows (see figure 2):

> *There is no way of ascertaining, by means of physical experiments performed in a closed laboratory, whether the laboratory is at rest in a uniform gravitational field* $\mathbf{g}$ *or is being uniformly accelerated with acceleration* $-\mathbf{g}$.

If this is so, we must have that (see figure 3):

> *There is no way of ascertaining, by means of physical experiments performed in a closed laboratory, whether the laboratory is falling freely in a uniform gravitational field or moving inertially—that is, with constant velocity—in free space.*

This second version of the principle is implicit in Corollary VI of Newton's Laws of Motion, which may be rendered thus:

> *If bodies, moved in any manner among themselves, are urged in the direction of parallel lines by equal accelerative forces, they will all continue to move among themselves, after the same manner as if they had not been urged by those forces.*[15]

* While explaining the Principle of Equivalence to the said Vienna assembly, Einstein observed that "Eötvös's experiment plays in this connection a role similar to that of Michelson's experiment in relation to the problem of the physical detectability of *uniform* motion" (ref. 9, p. 1255).

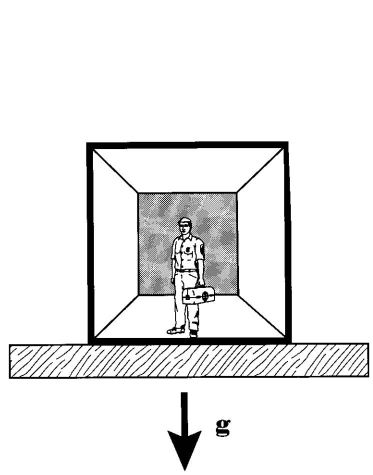

Fig. 2. A closed laboratory at rest in a uniform gravitational field **g** (left) and being uniformly accelerated with acceleration $-\mathbf{g}$ (right). Sketch by the author.

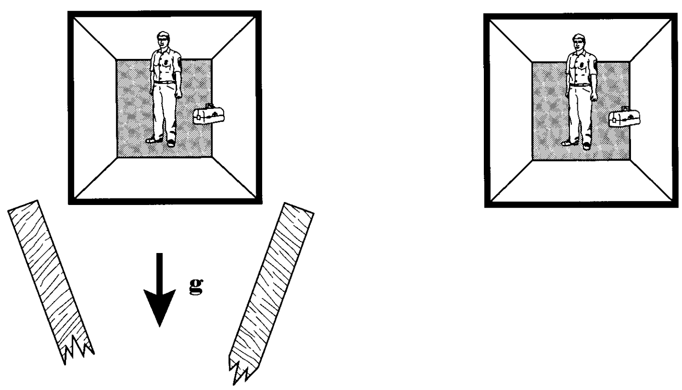

Fig. 3. A closed laboratory falling freely in a uniform gravitational field **g** (left) and moving inertially with constant velocity in free space (right). Sketch by the author.

It also flows directly from Einstein's story of how he first lighted on the principle, in the fall of 1907. It occurred to him that

> the gravitational field has only a relative existence in a way similar to the electric field generated by magnetoelectric induction. *Because for an observer falling freely from the roof of a house there exists*—at least in his immediate surroundings—*no gravitational field*. Indeed, if the observer drops some bodies then these remain relative to him in a state of rest . . . independent of their particular chemical or physical nature . . . . The observer therefore has the right to interpret his own state as "at rest."[16]

Einstein's fantasy of a freely falling observer agrees exactly with the now familiar sight of astronauts in orbit. Regardless of its composition, a body falling freely in a uniform gravitational field shows no sign of strain or stress, but hovers in a state of weightless bliss, just as it would if it were moving inertially, at an infinite distance from every source of gravity.

From the Equivalence Principle and the Principle of Energy Conservation Einstein promptly inferred that light gravitates and that natural clocks must appear to go more slowly when placed at a higher gravitational potential.[17] Both phenomena have been subsequently observed. Charles W. Misner, Kip S. Thorne and John Archibald Wheeler[18] argue—after A. Schild, they say—that the gravitational retardation of clocks precludes spacetime from being Minkowskian, and hence flat, so that the presence even of a uniform gravitational field betokens a non-zero spacetime curvature. I do not think their argument is right, but it is instructive. I shall sketch it, and then state my objection. Consider a source of light at rest in a uniform gravitational field. Every second a light pulse is emitted vertically in the direction in which the gravitational potential increases. The arrival of the light pulses is recorded at a point higher up, also fixed in the field. Owing to the gravitational retardation of clocks the pulses arrive at intervals longer than one second. Let us designate, respectively, by *A* and *B* the emission and reception of a given pulse, by *C* and *D* the reception and emission of the next one (figure 4). Events *A*, *B*, *C*, and *D* stand therefore at the vertices of a parallellogram in spacetime, with *unequal* opposite sides *AC* and *BD*. Such a parallellogram cannot exist in flat spacetime. Therefore, the spacetime underlying a uniform gravitational field is curved. The argument holds also if the gravitational field deflects light, provided that the successive light pulses describe spacetime trajectories that are curved in the same way (figure 4, right); for in this case the spacetime points *A*, *B*, *C*, and *D* still are the vertices of a parallellogram with unequal opposite sides. On the other hand—and this is the ground of my objection—the argument does assume that the spacetime geometry is reflected by ideally correct measurements performed with rigid rods and clocks, no matter what the state of motion of these tools. However, if the geometry is Minkowskian, it will be reflected by such measurements if and only if the rods and clocks are affixed to an inertial frame. Since that is not the case under the conditions prescribed for our thought-experiment, its result does not entail that the spacetime underlying a uniform gravitational field is curved, rather than Minkowskian. The argument does show, however,

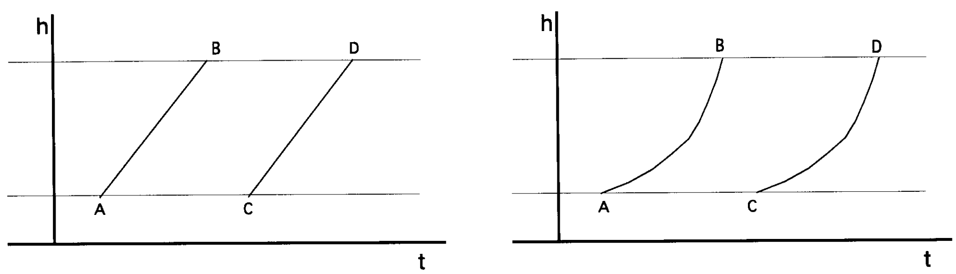

Fig. 4. The emission *A* and the reception *B* of one pulse of light and the reception *C* and emission of *D* of the next one, without (left) and with (right) gravitational deflection. Sketch by the author, after Misner, Thorne, and Wheeler, *Gravitation* (ref. 18), Figure 7.1, p. 188.

that spacetime coordinate systems defined in the usual way in a frame at rest on the surface of the earth cannot have the metrical import traditionally accorded to them. Einstein's research on gravity was delayed—he tells us—by his reluctance to accept this. For, as he observes in his *Autobiographical Notes*, "it is not easy to free oneself from the idea that the coordinates must have an immediate physical meaning."[19]

Eventually, however, Einstein discarded the traditional requirement that physical coordinates convey geometrical and chronometrical information. In a paper of 1916, he expressly associates this important step with his analysis of measurements carried out on a rotating disk with rods and clocks affixed to it.[20] He had already referred to it, in passing, in a paper of 1912. About the same time he must have taken another, more momentous step, which in his writings usually appears linked—some would say confused—with the former; namely, the decision to represent the gravitational field on a spacetime that is assumed to be locally—or rather tangentially—Minkowskian but whose global metric is not postulated *a priori* but must be read from gravitational phenomena. This decision cannot result from the Equivalence Principle alone, but surely rests on it taken together with the actual behavior of existing gravitational fields.

Consider a metrically significant coordinate system of the kind normally employed in special relativity. Such a system consists of Cartesian space coordinates defined by rigid rods at rest in an inertial frame and a time coordinate defined by clocks placed all over that frame and synchronized by Einstein's method of bouncing light signals. We choose units of length and time such that the vacuum speed of light $c = 1$.* A spacetime coordinate system meeting these conditions I call a *Lorentz chart*. The Minkowskian spacetime of special relativity admits global Lorentz charts precisely because its curvature is zero everywhere. In the light of the Equivalence Principle, second version, it is clear that a global Lorentz chart can be defined over a globally uniform gravitational field. This implies that the Misner-Thorne-Wheeler argument for non-zero curvature cannot be valid. But it also yields a much simpler argument for curved spacetime in the real world. For real gravitational fields are only locally and approximately uniform. As we all know, the direction of the field makes a full 360° rotation in space as one moves around the earth. Thus, the spacetime we live in does not admit a global Lorentz chart as a Minkowskian spacetime would. Therefore the spacetime we live in is not flat.

The argument suggests that there is some mysterious bond between spacetime metric and gravity, but, being purely negative and formal, it throws no light on this bond. It was, I think, mainly at this point that Einstein's incredible inventiveness came into play—or, if you prefer, his capacity for seeing affinities and analogies that did not occur to others. In special relativity there is an obvious connection between spacetime geometry and inertial motion: In the absence of external forces the spacetime trajectory or worldline of an ordinary massive particle does not change its spacetime direction; whence the coordinates assigned to such a free

* For example, choose the meter as the unit of length and put the unit of time equal to 1/299,792,458 second; or choose the second as the unit of time and the light-second as unit of length (1 light-second = the distance travelled by light in vacuo in one second = 299,792,458 meters, exactly).

particle by a Lorentz chart satisfy a linear equation, and the proper graphic representation of its life-history is a straight line. This can be expressed more precisely as follows: At any instant of its life-history a massive particle has a definite spacetime velocity, represented by the vector tangent to its worldline at that spacetime point;* if the particle is not subject to external forces, this vector remains constant in size and parallel to itself along the particle's worldline. One might say that the particle, when and where there are no force fields pulling it one way or the other, turns to the spacetime geometry for guidance. Einstein's daring thought conceives gravitation on this analogy. He had established the equivalence of inertial motion with free fall in a uniform gravitational field. He had long sympathized with Mach's somewhat hazy suggestion that inertia, no less than gravity, is due to the distribution of matter in the universe. He now saw a chance of putting this suggestion into an intelligible, workable shape. He resolved to treat the spacetime geometry as the sole source of guidance for freely falling particles in any gravitational field whatsoever, and to make the factual configuration of that geometry depend on the universal distribution of matter.

Einstein moved from Prague back to Zurich in August 1912. He later remembered having first perceived at that time the analogy between the mathematical problem associated with the theory of gravity he was reaching for and the theory of curved surfaces developed by Carl Friedrich Gauss.[21] Gauss produced a way of describing geometrical relations on an arbitrary surface intrinsically, that is, in terms of the lines and points that lie on the surface, without referring to the space outside it. This includes intrinsic notions of distance and curvature, and differential equations for the straightest lines (or *geodesics*) on the surface. The intrinsic geometry of a surface with non-zero Gaussian curvature is, of course, non-Euclidean, but every point of the surface has a neighborhood on which the surface geometry agrees well—to first order—with Euclidean plane geometry. This accounts for the fact that, for instance, midtown Manhattan appears to be divided into rectangular blocks, although no true rectangle can be drawn on the curved surface of our planet. Analogously, Einstein could account for the success of special relativity in our local laboratories by assuming that, on a suitable neighborhood of each point-event, the spacetime geometry agrees to first order with the flat Minkowskian geometry. This implies that spacetime is a Riemannian manifold of the same signature as Minkowski spacetime.** By virtue of it, Minkowski's classifi-

* The concept of a vector tangent to a surface at any given point is fairly obvious. Mathematicians have extended this notion to manifolds of more than two dimensions. For a tolerably precise yet —I hope— easily accessible presentation of one way of doing it, see Roberto Torretti, *The Philosophy of Physics* (Cambridge: Cambridge University Press, 1999), pp. 159-161.

** Some authors prefer to call such a manifold "pseudo-Riemannian" or "semi-Riemannian," because the metric is indefinite (the inner product of a non-zero tangent vector by itself can be positive, negative, or zero), and Riemann himself only contemplated manifolds with a positive definite metric. However, general relativity remains, to this day, by far the most significant physical application of Riemann's geometrical ideas. Since Riemann developed these ideas with a view to their eventual use in the description of natural phenomena, it is, in my view, particularly unfair to join his name, *in this connection*, with a disparaging prefix like "semi-" or "pseudo-."

cation of curves into timelike, spacelike and null is straightforwardly extended to curved spacetime. The analogy between inertial motion and free fall is understood to imply that freely falling particles follow timelike geodesics—in other words, that their spacetime velocities remain parallel to themselves along their respective worldlines—or, in still other words, that freely falling particles suffer no spacetime acceleration at all. In Riemannian manifolds the rate at which neighboring geodesics tend to converge toward—or to diverge from—each other depends directly on the local curvature.* Thus the apparent mutual attraction of nearby particles under the influence of gravity turns out to be, on this view, a manifestation of spacetime curvature. So, from Einstein's new standpoint, Newton's old problem about the cause of gravity is automatically solved, or rather, dissolved: Newton was unable to find the cause of gravity because there is no such thing. Free fall, just like inertial motion, is unconstrained, not the outcome of agency.

In August 1912 Einstein knew, of course, about Bernhard Riemann's generalization of Gauss's methods to spaces of any finite dimension.[22] He soon learned from his friend Marcel Grossmann that the tool required for doing mathematical physics on an arbitrary Riemannian manifold was available in the guise of the absolute differential calculus.[23] Wielding this powerful tool, Einstein and Grossmann produced "a generalized theory of relativity and a theory of gravity,"[24] which agrees with general relativity in intent and in many basic features. The main ideas common to both theories can be summed up as follows:

(A) Spacetime is a four-dimensional Riemannian manifold whose metric agrees to first order with the Minkowski metric on a neighborhood of every point. Natural clocks measure proper time along their respective worldlines.
(B) The metric field guides free fall, just as the Minkowski metric guides inertial motion in special relativity: An ordinary massive particle that has no angular momentum and is not subject to any external influence except that of gravity describes a timelike geodesic. Photons describe null geodesics.
(C) The metric field is therefore none other than the gravitational field, and therefore depends on the distribution of matter and non-gravitational energy in spacetime.
(D) The dependence of the metric field on matter-energy is governed by the gravitational field equations, a set of second order differential equations in the metric components that are designed to agree with the Poisson equation of Newtonian theory in the limiting case of slow motion in a weak field.

Thus, for the reasons stated, Einstein was already thinking of gravity as curved spacetime in 1913. Still missing was a satisfactory system of differential equations linking the spacetime curvature to the distribution of matter. The field equations of general relativity were published on November 25, 1915 (ref. 1). Since there are

* The dependence is governed by the equation of geodesic deviation. This is derived and explained, for instance, in Ignazio Ciufolini and John Archibald Wheeler, *Gravitation and Inertia* (Princeton: Princeton University Press, 1995), pp. 31–36; also, more concisely, in Robert M. Wald, *General Relativity* (Chicago: University of Chicago Press, 1984), pp. 46–47, and in Bernard F. Schutz, *Geometrical Methods of Mathematical Physics* (Cambridge: Cambridge University Press, 1980), pp. 213–214.

compelling reasons for regarding them as the one and only viable solution to the mathematical problem raised in point (C), many have wondered why Einstein did not find them right away, but spent much time and effort arguing for the Einstein-Grossmann equations. But this is not a question for us to deal with now.

Instead, I shall consider a purely conceptual question that has been around almost as long as general relativity itself. This question is usually ignored in relativity textbooks, but I have seen it put in an appearance in at least two recent philosophical publications, one of which[25] comments extensively on the book by Richard P. Feynman (ref. 29) discussed below, while the other[26] approvingly quotes a paper by Sir Harold Jeffreys (ref. 27), who raised the question as early as 1919, when the solar eclipse observations of Eddington and his associates allegedly confirmed Einstein's theory of gravity.

For simplicity's sake, let us assume that the phenomena that contemporary physics calls gravitational form a natural kind and that the Einstein field equations epitomize our knowledge of such phenomena in the best way that is currently available. In these equations the unknowns are 10 scalar functions, which are the possibly distinct components, relative to a particular spacetime chart, of a symmetric covariant tensor field of rank 2. From a mathematical point of view, such a tensor field does not differ at all from a Riemannian metric tensor that agrees locally to first order with that of Minkowski spacetime. But, one may ask, does this mathematical analogy entail that *a solution of the Einstein field equations* obtained under realistic physical assumptions actually characterizes the *geometry of the world*? This is the conceptual question I wish to examine. I doubt that anyone would have thought of asking it if she or he had in mind the process I have sketched, by which the Einstein field equations were found not as a set of black marks on white paper, but as the appropriate symbolic expression of a certain way of thinking (which, as we saw, was thoroughly steeped in geometry). But philosophers do not always pay attention to processes of this sort, which do not belong to the epistemically lofty "context of justification" but to the lowly, contingent "context of discovery." And scientific research, of course, is generally oblivious of them.

Jeffreys's reading of general relativity in his paper of 1919[27] is decidedly *ageometretos*. He does, indeed, characterize the worldlines of the freely falling particles as being the extremals of a certain integral, whose integrand ds is defined, like a Riemannian metric, by the equation

$$\mathrm{d}s^2\sum_i \sum_j g_{ij}\mathrm{d}x_i\mathrm{d}x_j \qquad (2)$$

where the g_{ij}'s are certain functions of the coordinates. But Jeffreys expressly warns us that

> No assumption is made concerning the identity of ds with the line element in the 4-dimensional space-time, or connecting the g_{ij}'s with the "curvature" of such space-time. *Space*, *time*, *and curvature are metaphysical conceptions whose utility*, *if any*, *depends wholly on the individual peculiarities of the person trying to understand them*. The quantities are those derived from physical measurements, so that the invariance of ds is no longer axiomatic.[28]

Thus, according to Jeffreys, the integrand ds of the integral that is extremalized along the worldlines of freely falling particles has nothing to do with the geometry—or chronogeometry—of the world.*

Richard Feynman's ungeometric reading of Einstein's theory of gravity is more enlightened and therefore a good deal more interesting than Jeffreys's.[29] Feynman toys with the idea that one could proceed from a classical to a quantum field theory of gravity more or less in the same way in which Sin-itiro Tomonaga, Julian Schwinger and himself successfully went from classical to quantum electrodynamics.[30] In his first lecture he proposes the following exercise in history-of-science fiction: Physicists on Venus are well acquainted with 30 different physical fields and have developed suitable theories for them, but only recently have they discovered gravity. They naturally conceive it as the 31st force field, on the analogy of the other 30. Feynman discusses what kind of particle will be associated with this field. From the familiar features of gravitational phenomena —especially those expressed by the equivalence principle and the inverse square law—he infers that this particle cannot be one of those already known to us, in particular, not the neutrino, but must be a hitherto unknown particle of spin 2, which he calls the *graviton*. The field associated with a spin-2 particle must be described by a symmetric rank-2 tensor potential I shall denote by **h**. Following the analogy of quantum electrodynamics, the tensor field **h** is defined on "the space of special relativity,"[31] that is to say, on Minkowski spacetime. As we know, the metric structure of Minkowski spacetime is given by another symmetric rank-2 tensor field, usually denoted by $\boldsymbol{\eta}$. Feynman sketches a quantum theory of the gravitational field **h**. To compute its macroscopic effects, for example, the orbits of planets circulating around a star, this quantum theory must be reduced to its classical form. This is done by deriving the classical theory from a variational principle that involves a time integral of the Lagrangian. By an elegant argument, Feynman derives the following formula for this integral:

$$m_0\left[-\frac{1}{2}\int \mathrm{d}\tau\, g_{\mu\nu}(x)\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\frac{\mathrm{d}x^\nu}{\mathrm{d}\tau}\right] \tag{3}$$

where τ is proper time and the $g_{\mu\nu}$ are the components, relative to chart x, of a symmetric rank-2 tensor field **g** introduced, for compactness in writing, by the following definition:

$$\mathbf{g}(x) = \boldsymbol{\eta} + 2\mathbf{h}(x)\sqrt{8\pi G} \tag{4}$$

(where G is the usual gravitational constant in natural units $\hbar = c = 1$).**

* Jeffreys is well aware of the formal relationship between the integrand ds defined by (4) and the line element of Minkowskian spacetime: "The condition that ds^2 in its most general form can be transformed into the form $c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$ can be shown to be that all the 96 components of a certain vector, the Riemann-Christoffel tensor, shall vanish" (ref. 27, p. 147). But in his paper there is not the slightest hint that the Riemann-Christoffel tensor might have anything to do with the "metaphysical" notion of curvature.

** It is worth noting that Feynman does not write this definition as I do, but in terms of components, thus:

Feynman has little difficulty in proving that the tensor **g** must satisfy the Einstein field equations. Does this mean that the classical approximation to Feynman's Venusian theory of gravity is equivalent to general relativity? Feynman apparently thought so. He presents Einstein's geometric reading of the theory as an alternative "interpretation" of the "physical" field theory he ascribes to his Venusians.

> Since these are truly two aspects of the same theory we might assume that the Venutian [*sic*] scientists, after developing their completed field theory of gravity, would have eventually discovered the geometrical point of view. We cannot be absolutely sure, since one cannot ever explain inductive reasoning—one cannot ever explain how to proceed, when one knows only a little, in order to learn even more. In any case, the fact is that a spin-two field has this geometrical interpretation; this is not something readily explainable—it is just marvelous.[32]

Feynman believes that "the geometrical interpretation is not really necessary or essential to physics."[33] Nonetheless, he finds it quite attractive in one respect. He reminds us that his initial point of view "was that space is describable as the space of special relativity," which, for some reason that eludes me, he calls "Galilean."

> In this Galilean space, there might be gravity fields $\eta_{\mu\nu}$ which have the effect that rulers are changed in length, and clocks go at faster or slower rates. So that in speaking of the results of experiments we are forced to make distinctions between the scales of actual measurements, physical scales, and the scales in which the theory is written, the Galilean scales. Now, the point is that it is the physical coordinates that must always reproduce the same results. It may be convenient in order to write a theory in the beginning to assume that measurements are made in a space that is in principle Galilean, but after we get through predicting real effects, we see that the Galilean space has no significance. . . . Therefore, we see that it might be a philosophical improvement if we could state our theory from the beginning in such a way that there is no Galilean space that enters into the specification of the physics; we always deal directly with the physical space of actual measurements.[34]

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + 2h_{\mu\nu}(x)\sqrt{8\pi G} \tag{5}$$

Normally, one should not make much of this difference in writing, but in this particular case we have that in the literature on relativity the $\eta_{\mu\nu}$ stand for the components of the Minkowski metric relative to a Lorentz chart:

$$\eta_{\mu\nu} = \begin{cases} 1 \text{ if } \mu = \nu = 0 \\ -1 \text{ if } \mu = \nu = 1, 2, \text{ or } 3 \\ 0 \text{ if } \mu \neq \nu \end{cases} \tag{6}$$

If $\eta_{\mu\nu}$ in eqn. (5) has its standard meaning, then one must assume that $g_{\mu\nu}$ and $h_{\mu\nu}$ also stand for tensor components relative to a Lorentz chart. If you are acquainted with Einstein's thought on these matters you cannot help asking yourself how a Lorentz chart can be set up in a realistic gravitational field, given that, by definition, such a chart must be adapted to an inertial reference frame, and therefore, by the equivalence principle, it can be defined on a gravitational field only if the latter is globally uniform. This irksome question is avoided by using the coordinate-free notation of eqn. (4).

Feynman's words of approval for the geometrical interpretation of the theory of gravity ought not to blind us to the limitations that his approach imposes on it. If the solution **g** of the Einstein field equations is required to satisfy condition (4), where **η** is the metric tensor of "the space of special relativity" and **h** is a tensor field defined on this space, the manifold underlying all three fields **η**, **h**, and **g** must be such that the flat metric **η** can be defined on it. This requirement places severe restrictions on the admissible manifold structure. Usually, "the space of special relativity" is considered to be homeomorphic—*i.e.*, topologically equivalent—to $\mathbb{R}^4$ (that is, the collection of all real number 4-tuples, with the standard topology generated by the open balls). This is not strictly necessary, and one can generate flat spacetimes of very different shapes by mutually identifying certain sets of points, just as one generates the metrically flat surfaces of a cylinder or a Moebius strip by identifying opposite sides of a rectangle.* But such variety is confined within very narrow bounds and is as nothing compared to the staggering diversity of topologically incompatible spacetimes allowed by general relativity as conceived by Einstein. Thus, Feynman's Venusians could very well conceive the axiosymmetric spacetime of the earliest exact solution of Einstein's field equations (the Schwarzschild solution)[35] as a Minkowski spacetime punctured along a timelike straight line and warped by the **h** field in the surroundings of this line. But they would have to exclude, on topological grounds, the exact solution that paved the way for modern cosmology when Einstein found it in 1917,[36] for the flat metric **η** simply cannot exist on the manifold—shaped like a hypercylinder, homeomorphic to $S^3 \times \mathbb{R}^1$—that is required by this solution. Nor could they countenance the expanding-and-contracting world models of Alexander Friedmann[37] and Georges Lemaître.[38] As is well known, these spatially closed "Big Bang" models have been more popular in relativistic cosmology than the spatially open, ever expanding ones,[39] despite that observational astronomy has persistently failed to record the high average density of matter that would be required to bring about the contraction stage. The Venusian physicists would be spared this uncertainty. If a Venusian mathematician discovered the Friedmann solutions of the Einstein field equations, the spatially closed ones would at once be discarded as physically impossible—for they cannot be realized on "the space of special relativity"—and only the spatially open ones would be regarded as viable. Moreover, such a mathematical discovery could not be prompted, like Friedmann's, by physically and philosophically motivated "cosmological considerations" *à la* Einstein, for, in Feynman's Venus, Einstein's finite yet limitless relativistic world model (ref. 36) could not come up for discussion.

Such speculations about alternative developments of physics on different planets are no doubt highly questionable. Still, it may be worth remembering, in the present context, that all the more forceful evidence hitherto gathered for general relativity rests squarely on applications of the Schwarzschild solution (precession of perihelia,

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* For example, "identifying the point (x^1, x^2, x^3, x^4) with the point $(x^1, x^2, x^3, x^4 + c)$, where c is a constant, changes the topological structure from $\mathbb{R}^4$ to $\mathbb{R}^3 \times S^1$, and introduces closed timelike lines into the space–time" (S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space–Time*, (Cambridge: At the University Press, 1973), p. 124).

deflection of light by gravity) or the Equivalence Principle (gravitational redshift). Observational evidence for "Big Bang" cosmology—the Hubble recession of galaxies,[40] the Penzias-Wilson radiation background[41]—can hardly seem cogent, unless it is combined with the acceptance of general relativity based on solar system data (read in the light of the Schwarzschild solution). Therefore, it would be rash to say that Feynman's ungeometrical approach to Einstein's theory is undermined by modern cosmology and its characteristic demand for topological freedom. Indeed, it is not clear that observations and experiments conducted by human beings on earth can warrant the ascription of out-of-the-way topologies to the world as a whole.

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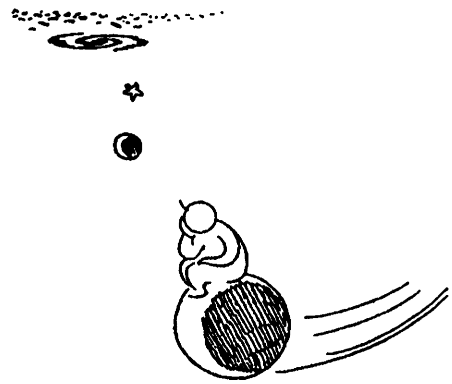

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